# Premières preuves empiriques de chaos dans les ventes de biens à la mode
# First empirical evidence of chaos in the sales of fashion goods*

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Cahier du FARGO n° 1110602
Version - juin 2011

*Résumé* : L'objectif de cet article est de tester la présence de chaos dans les ventes de biens à la mode. Après avoir brièvement rappelé le débat théorique autour de cette hypothèse, nous présentons les résultats. Ces résultats renforcent la plausibilité de l'existence d'un comportement chaotique des ventes de biens à la mode. Cependant, au fil du temps, les ventes étudiées semblent être de moins en moins chaotiques.

*Mots clés* : chaos déterministe ; mode ; vente ; exposant de Lyapunov ; non-linéarité.

*Abstract*: The purpose of this study is to test the hypothesis that the sales of fashion goods are chaotic. After reviewing briefly the theoretical debate on this hypothesis, we present our results, which provide support for chaotic patterns in the sales of fashion goods. However, the sales studied seem to become less chaotic over time.

*Key words*: deterministic chaos; fashion; sales; Lyapunov exponent; nonlinearity.

*JEL Classification* : C22 ; C52 ; D11 ; L81

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[*] We gratefully acknowledge Michelle Assay Eshghpour for her helpful comments on an earlier draft.

## 1. Introduction

“Although the mathematics of new product diffusion models clearly allow for chaotic bifurcations and fluctuations, these phenomena have not been reliably observed for actual products” (Phillips and Kim, 1996: 239). To the best of our knowledge, this statement is still valid today. However, there is theoretical debate, which remains to be addressed with empirical data. To address this debate, at least one long time series of sales is needed: “The mathematical tests for empirical chaos are sophisticated, and require hundreds of observations as input. Few marketing data sets of interest contain such a volume of observations” (Phillips and Kim, 1996: 241). A chaotic system can be intuitively defined as a deterministic dynamic system that is sensitive to initial conditions and thus cannot be predicted in the long run. “Sensitive to initial conditions” means that two very close initial conditions will produce two trajectories that will diverge exponentially. This exponential divergence is measured by the Lyapunov exponent.

The addressed debate is the following one. Granovetter and Soong's (1986) model shows that the sales of fashion goods can be chaotic. Fashion can be defined as “a way of behaving that is temporarily adopted by a discernible proportion of members of a social group because that chosen behavior is perceived to be socially appropriate for the time and situation.” (Sproles, 1979; cited by Miller et al., 1993: 143) Thus, fashion is characterized by a bandwagon effect: “the extent to which the demand for a commodity is increased due to the fact that others are also consuming the same commodity.” (Leibenstein, 1950: 189) This effect arises for children's goods, like Barbie dolls or video game hardware (Pindyck and Rubinfeld, 2008). More recently to represent these sales, Nakayama and Nakamura (2004) have also introduced a nonlinear model that exhibits chaotic patterns. However, by introducing interpersonal effects at the preference level in a market of rational consumers, Iannaccone (1989) has demonstrated that chaotic behaviors could be less possible than in Granovetter and Soong's model. Moreover, Miller et al. (1993) have built a linear model to account for the behavior of the sales of fashion goods. In such a model, simulated sales cannot be chaotic.

Despite the overwhelming theoretical attention to the phenomenon of chaos in the sales of fashion goods, it however lacks empirical evidence. Thus, the aim of this study is to test for chaos in the sales of fashion goods. To accomplish this aim, the paper is organized as follows. Section 2 describes the data and the methods used. Section 3 reports the results. Section 4 concludes.

## 2. Material and methods

### *2.1. Data*

Following Pindyck and Rubinfeld (2008), we used weekly sales of video game hardware, which were found in the database of the website www.vgchartz.com. The interested reader is referred to http://www.vgchartz.com/methodology.php for more details on data collection.

To detect the presence of nonlinearities and chaos, long time series are required. Thus, we needed time series with at least 400 observations (Lin, 1997). Because the Japanese sales of Game Boy, Play Station, Play Station 2 and Super Nintendo are long enough, the following described methods were only used for these series.

### *2.2. Statistical analysis*

Following Benincà et al. (2008), these series were rescaled using a Box-Cox transformation because the original sales showed time-varying variance. Because the rescaled series showed time-varying mean, they were filtered using moving average with a bandwidth of 52 weeks. These transformations to stationarity were required to avoid bias in the estimation of the largest Lyapunov exponent (Benincà et al., 2008) and erroneous acceptance of the hypothesis of the presence of nonlinearities (Hsieh, 1991).

Then, Brock, Dechert and Scheinkman's test (BDS test, hereafter) was used to detect the presence of nonlinearities. To detect nonlinearities, this test was performed on the stationarized series, which were filtered for linear dependence using autoregressive filters. For each series, the order of the autoregressive filter was chosen by triangulation of the Akaïke, Schwarz, and Hannan-Quinn information criteria.

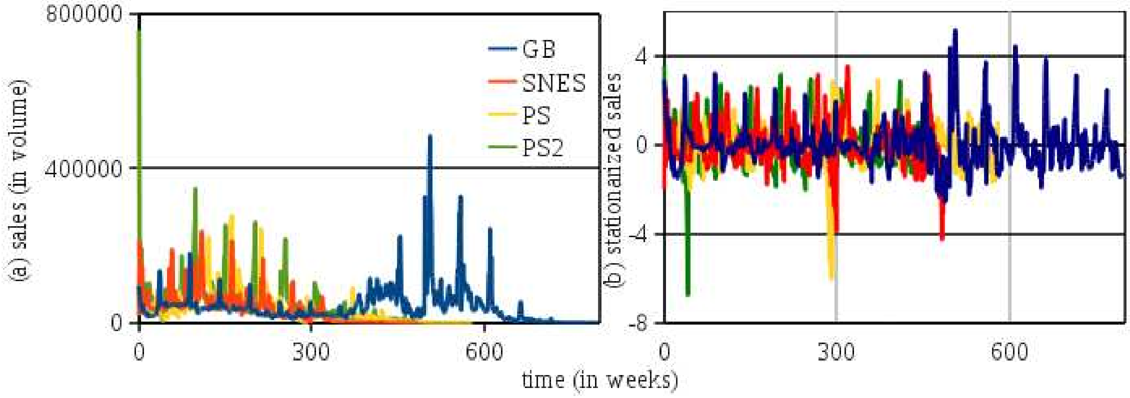
Because chaotic patterns can explain the nonlinearities detected (Granovetter and Soong, 1986; Nakayama and Nakamura, 2004), the presence of chaos was tested with an algorithm developed for short series (Rosenstein et al., 1993). This algorithm requires a suitable choice of embedding dimension and time delay. The time delay was chosen as the time lag where the autocorrelation function drops to 1-1/exp(1) (Rosenstein et al., 1993). The embedding dimension was estimated by the value for which the number of false nearest neighbors drops to zero (Kantz and Schreiber, 2004). Because the estimation of the largest Lyapunov exponent can be biased if nearby sales (in the state space) are also near in time, the Theiler window was used to remove temporally nearby sales from the set of pairs selected to estimate the largest Lyapunov exponent (Theiler, 1986). By visual inspection of space time separation plot (Provenzale et al., 1992), for each stationarized series, a suitable size of the Theiler window was estimated.

Finally, the result of Rosenstein et al.'s algorithm was used with the procedure described by Liu et al. (2005) because this procedure is robust to noise. This procedure was performed to evaluate the largest Lyapunov exponent, the predictability horizon and the level of noise for each series.

## 3. Results

Before transformation the sales studied show time-varying variance and mean (Figure 1(a)). After transformation these sales look like stationarized (figure 1(b)). ADF and KPSS tests were used to test for stationarity of the transformed series (available on request).

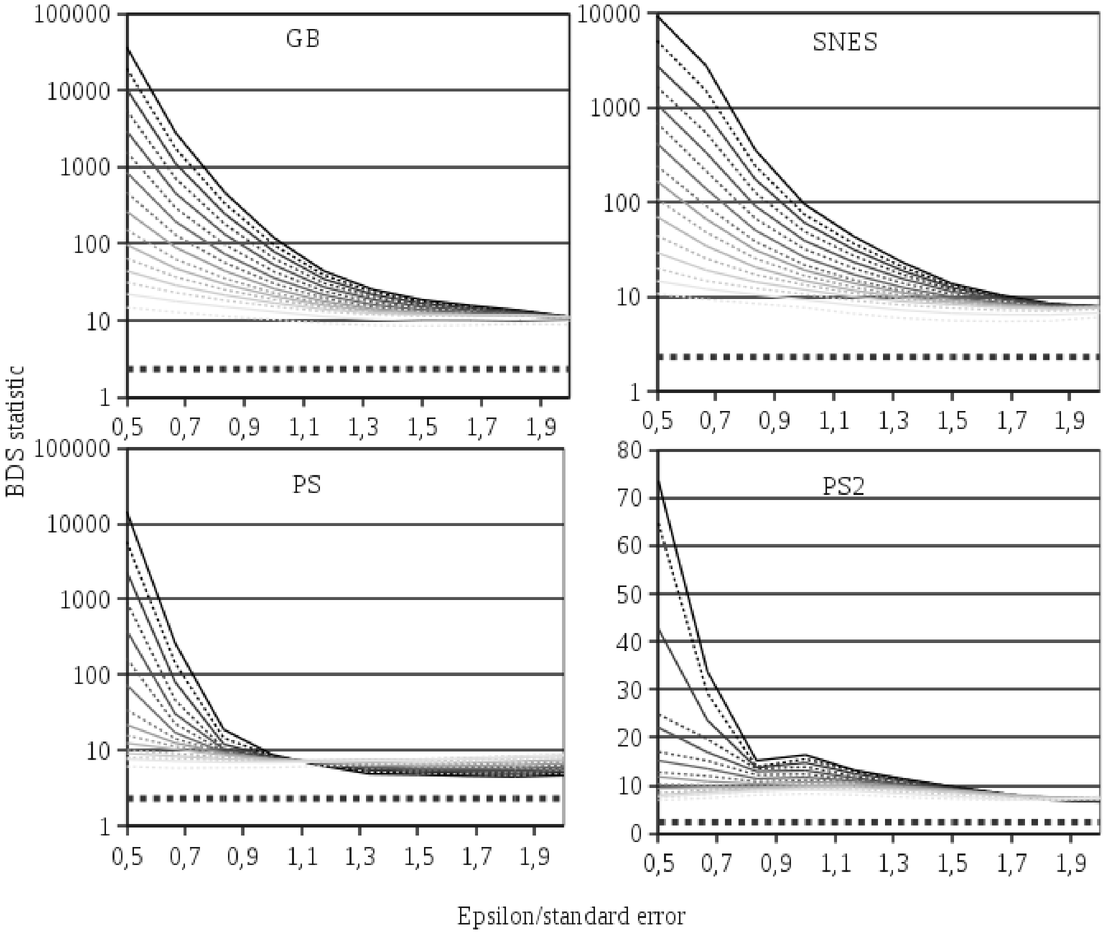
Figure 1
Original sales and sales transformed to stationarity



Note: Horizontal axis represents the time from the product launch for each series. Vertical axis represents (a) the sales of Game Boy (GB), Super Nintendo (SNES), Play Station (PS) and Play Station 2 (PS2) and (b) the sales stationarized.

These stationarized sales are significantly nonlinear (Figure 2). For all suitable values of epsilon and embedding dimensions, the BDS statistic is higher than the critical value. Are chaotic patterns responsible for these nonlinearities?

Figure 2
Results of BDS test on stationarized sales



Note: For the sales of Game Boy (GB), Play Station (PS) and Play Station 2 (PS2), the BDS statistic was estimated after using AR (1) filter. For the sales of Super Nintendo (SNES), this statistic was estimated after using AR (2) filter. To limit type I error, following Brock et al. (1996), epsilon is set to between half and four halves the standard deviation of the stationarized sales (horizontal axis), and the embedding dimension is set to between two (the lightest fine dashed curve) and seventeen (the darkest fine solid curve). The bold dashed curve represents the critical value at 1% level: z=2.32.

For each stationarized sales, the largest Lyapunov exponent is significantly positive (Table 1). By using the procedure described by Liu et al. (2005), the output of Rosenstein et al.'s (1993) algorithm permits the quantification of the noise level and of the predictability horizon.

Supposing that the horizon of predictability is the delay for which the logarithm of the exponential divergence reaches 95 % of its maximum, it is possible to evaluate this horizon for each series. The predictability of the sales of Game Boy, Super Nintendo, Play Station and Play Station 2 was limited to a time horizon of 6.4, 7.3, 7.8 and 8.7 weeks, respectively.

Table 1
Results of a robust to noise evaluation of the largest Lyapunov exponent

| | ***Intercept*** | ***Noise level*** | ***Largest Lyapunov exponent*** | ***P-value*** |
|---|---|---|---|---|
| ***St(GB)*** | -3.1 | 1.78 | 0.1611455*** | 0.001 |
| ***St(SNES)*** | -2.81 | 1.75 | 0.1587826*** | 0.002 |
| ***St(PS)*** | -3.06 | 1.68 | 0.1469196*** | 0.002 |
| ***St(PS2)*** | -3.19 | 1.63 | 0.1408065*** | 0.001 |

Note: This table presents the evaluation of the parameters of Liu et al.'s (2005) model. These estimations were performed with the first six points of the results of Rosenstein et al.'s (1993) algorithm for each stationarized series. St(.) denotes that the series were stationarized. The series studied are the Japanese sales of Game Boy (GB), Super Nintendo (SNES), Play Station (PS) and Play Station 2 (PS2). *** indicates that the largest Lyapunov exponent is significant at 1% level.

## 4. Conclusion

The results of the BDS test suggest the presence of nonlinearities in the sales studied. Thus, Miller et al.'s linear model seems to be insufficient to account for the sales studied. Evidence of chaos provides support for Granovetter and Soong's, and Nakayama and Nakamura's nonlinear models.

However, the horizon of predictability seems to increase over time. This result suggests that video game hardware becomes old-fashioned and thus its sales are less and less chaotic.

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